

# Extremal Branes as Elementary Particles

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## Abstract

The supersymmetric p-branes of Type II string theory can be interpreted after compactification as extremal black holes with zero entropy and infinite temperature. We show how the p-branes avoid this apparent, catastrophic instability by developing an infinite mass gap. Equivalently, these black holes behave like elementary particles: they are dressed by effective potentials that prevent absorption of impinging particles. In contrast, configurations with 2, 3, and 4 intersecting branes and their nonextremal extensions, behave increasingly like conventional black holes. These results extend and clarify earlier work by Holzhey and Wilczek in the context of four dimensional dilaton gravity.

## 1 Introduction

The recent interest in black holes has focussed on extremal configurations with finite area and their non-extremal generalizations because, in these cases, the finite entropy inferred from the area can be related to a microscopic counting in string theory [1] (for reviews see [2, 3]). Four dimensional black holes with finite area arise by compactification of configurations with at least 4 intersecting branes. However, there are also supersymmetric configurations with 3, 2 and 1 intersecting branes. The corresponding black holes all have vanishing area and their formal temperatures, defined from the surface gravity, are 0, finite and infinite respectively [4]. The nonzero temperatures naively indicate that the compactified supergravity solutions are semiclassically unstable since they would radiate to produce naked singularities. If we are to make sense of compactified p-brane solutions there must therefore be a mechanism that stabilizes these objects.

The analogous problem was confronted some time ago in the context of dilaton gravity with action:

$$S = \int d^4x \sqrt{-g} \left( R - 2(\nabla\phi)^2 + e^{-2a\phi} F^2 \right) \quad (1)$$

The extremal black hole solutions of this theory [5, 6] have non-zero entropy for  $a = 0$  but vanishing entropy for  $a > 0$ . Furthermore, the formal temperature is zero for  $a < 1$ , finite for  $a = 1$  and infinite for  $a > 1$ . The analogy with intersecting branes is precise because classical solutions to Eq. 1 with  $a = 0, 1/\sqrt{3}, 1, \sqrt{3}$  can be interpreted in the context of type II string theory as marginally bound states of elementary solutions with  $a = \sqrt{3}$  [7, 8, 9, 10]. These extremal black holes with  $a = \sqrt{3}$  played a crucial role in the duality revolution [11, 12] and have since been interpreted at weak coupling as D-branes [13]. Some of the required marginal bound states have been shown to exist [14, 15].

Extremal black holes with non-zero temperature inevitably develop naked singularities and are therefore not physically acceptable. Holzhey and Wilczek [16] discovered that black holes with  $a > 1$  have infinite mass-gaps - i.e., they support no finite energy excitations! This causes the thermal description to break down and moreover these black holes are unable to absorb any finite energy impinging objects. This implies that there is no radiation into these modes either, via Kirchoff's Law. In these senses, the  $a > 1$  holes act like elementary particles, rather than as black holes. For  $a = 1$  the mass gap is finite, leading to a situation where the black hole is totally repulsive for objects below a particular critical energy. This also implies suppression of radiation at energies below this bound. The finite temperature suggests that these black holes might develop naked singularities by radiating modes with energies higher than the gap. However, in [17] it was argued that the thermodynamic description breaks down close to extremality in such a way that this is avoided.

In this paper, we will exhibit these phenomena directly in the context of compactified p-brane solutions. Holzhey and Wilczek found that perturbations of the metric, dilaton and other fields displayed the same qualitative behaviors as a minimally coupled spectator scalar; so we limit ourselves to the latter case. The field equation governing such a scalar is remarkably simple, despite a very general background. Indeed, it reduces to the Schrödinger equation for a quantum particle in an attractive potential. Excitation of the black hole corresponds to absorption by the potential and repulsion of impinging particles below a critical energy implies a gap in the excitation spectrum. As discussed above, the gap also implies a breakdown in the thermal description of the object. In our approach the crucial distinction between black holes with  $a > 1$  and  $a < 1$  arises as a simple consequence of the well-known feature of  $\frac{1}{r^s}$  potentials that they capture particles if  $s > 2$  but not if  $s < 2$ .

The paper is organized as follows. In section 2, we write the equations of motion for a spectator scalar, minimally coupled to the ten-dimensional Einstein metric. We first consider a single extremal p-brane and then several intersecting ones. In section 3 we toroidally compactify intersecting branes to make four-dimensional black holes and exhibit the repulsive properties (or mass gaps) of one and two intersecting branes and the absorption by three or four intersecting branes. In Section 4 we discuss

scattering from non-extremal p-branes and the approach to extremality. The general wave equation acquires a particularly simple form. Finally, in Section 5, we derive the temperatures of uncompactified extremal p-branes in 10 dimensions and show that they too develop mass-gaps of the order of their temperatures.

## 2 Effective Potentials

### 2.1 Extremal p-Branes

The 10 dimensional form of the type II action is

$$S = \int d^{10}x \sqrt{-g_S} \left[ e^{-2\phi} (R_S + 4(\nabla\phi)^2) - \frac{2}{(p+2)!} F_{p+2}^2 \right] \quad (2)$$

where  $F$  is an RR (p+2)-form field strength and the subscripts  $S$  indicate that the string metric is being used. The classical theory features extended p-brane solutions that are sources for the field strengths in the action [18].<sup>1</sup> Their extremal incarnations are:

$$ds_S^2 = D^{-1/2} (-dt^2 + dx^i dx_i) + D^{1/2} (dy^2 + y^2 d\Omega_{8-p}^2) \quad (3)$$

$$e^{-2\phi} = D^{(p-3)/2} \quad (4)$$

$$F = \frac{Q}{y^{8-p}} D^{-2} dt \wedge dx_1 \wedge \cdots \wedge dx_p \wedge dy \quad (5)$$

$$D = 1 + \left( \frac{2}{7-p} \right) \frac{Q}{y^{7-p}} \equiv \left( 1 + \frac{Q_p}{y^{7-p}} \right) \quad (6)$$

The index  $i$  runs from 1 to  $p$  and spans the brane volume. In the Einstein metric, related to the the string metric through  $g_{S\mu\nu} = g_{E\mu\nu} e^{\phi/2}$  [19], the metric Eq. 3 becomes

$$ds_E^2 = D^{(p-7)/8} (-dt^2 + dx^i dx_i) + D^{(p+1)/8} (dy^2 + y^2 d\Omega_{8-p}^2) \quad (7)$$

Note that the transverse part is conformally flat, with conformal factor  $C^2 = D^{\frac{p+1}{8}}$ , in the isotropic coordinates employed here. Now consider a spectator scalar field that is minimally coupled to the background Einstein metric of Eq. 7. The equation of motion

$$D_\mu \partial^\mu \chi = \frac{1}{\sqrt{-g_E}} \partial_\mu (\sqrt{-g_E} g_E^{\mu\nu} \partial_\nu \chi) = 0 \quad (8)$$

simplifies because the volume element

$$\sqrt{-g_E} = \left[ D^{\frac{p+1}{8}(9-p)} D^{\frac{p-7}{8}(p+1)} \right]^{\frac{1}{2}} = D^{\frac{p+1}{8}} \quad (9)$$

is identical to the conformal factor of the transverse space  $C^2 = g_{yy}$ . Eq. 8 acquires the simple form:

$$[D \square_{p+1} + \triangle_{9-p}] \chi = 0 \quad (10)$$

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<sup>1</sup>See also [19, 20] for the solutions of [18] in the isotropic coordinates used here.

where  $\square_{p+1}$  is the Klein-Gordon operator (with Lorentzian signature) in the  $p+1$  dimensional world volume theory and  $\Delta_{9-p}$  is the Laplacian (with Euclidian signature) in the directions orthogonal to the brane. The crucial simplification is that both are operators in *flat* space.

Introducing the spatial coordinates  $\vec{x}$  and  $\vec{y}$  that are tangent and transverse to the brane, respectively, and partially Fourier transforming as  $\chi = \exp i(\omega t \pm \vec{k} \cdot \vec{x})\psi(\vec{y})$  the expression becomes

$$\left(-\Delta_{9-p} - \frac{(\omega^2 - \vec{k}^2)Q_p}{y^{7-p}}\right)\psi = (\omega^2 - \vec{k}^2)\psi \quad (11)$$

This is simply a Schrödinger equation for a Coulomb problem in  $9-p$  dimensions. There is therefore a wealth of results that can be drawn upon when analyzing the dynamics. However, before doing so, we will show that analogous simplifications occur for more general brane configurations.

## 2.2 Intersecting Extremal Branes

The extremal branes can be viewed as building blocks that can combine into extremal, intersecting configurations that are also classical solutions of the theory in Eq. 2. This class of solutions includes particularly interesting ones that can be interpreted as supersymmetric black holes with finite area after toroidal compactification [4, 21, 22, 23]. Intersecting extremal branes can be constructed as follows. Extremality is equivalent to preserving supersymmetry which in turn implies that p- and q-branes orthogonally intersecting on a k-brane must satisfy  $p+q-2k \equiv 0 \pmod{4}$  [24]. In this case intersecting brane solutions are given by the “harmonic function rule” [25, 26]: multiply the harmonic functions associated with individual p-branes in Eq. 7 for each metric component independently and similarly for the dilaton. For example, the intersection of a p-brane wrapped around the  $(1, \dots, p)$  dimensions and another wrapped around the  $(3, \dots, p+2)$  dimensions gives rise to the fields

$$\begin{aligned} ds_E^2 = & (D_1 D_2)^{\frac{p-7}{8}} \left(-dt^2 + dx_3^2 + \dots dx_p^2\right) + D_1^{\frac{p-7}{8}} D_2^{\frac{p+1}{8}} \left(dx_1^2 + dx_2^2\right) \\ & + D_1^{\frac{p+1}{8}} D_2^{\frac{p-7}{8}} \left(dx_{p+1}^2 + dx_{p+2}^2\right) + (D_1 D_2)^{\frac{p+1}{8}} \left(dx_{p+3}^2 + \dots dx_9^2\right) \end{aligned} \quad (12)$$

$$e^{-2\phi} = (D_1 D_2)^{(p-3)/2} \quad (13)$$

where  $D_1$  and  $D_2$  are harmonic functions of the directions transverse to both branes, *i.e.*  $(p+3, \dots, 9)$ . The corresponding expression in string metric is related to Eq. 12 by the factor  $e^{\phi/2} = (D_1 D_2)^{-(p-3)/8}$ .

In general, the Einstein metric for the dimensions parallel to the  $i$ th brane is multiplied by  $D_i^{\frac{p-7}{8}}$  and the dimensions perpendicular to the brane are multiplied by  $D_i^{\frac{p+8}{8}}$ . The harmonic function is  $D_i = 1 + Q_i/r^{s-2}$  where  $s$  is the number of dimensions transverse to all the branes and  $r$  is the coordinate radius in these dimensions. Finally, the expression for the dilaton is simply  $e^{-2\phi} = \prod_i D_i^{(p_i-3)/2}$ .

Consider a scalar field  $\chi$  coupled minimally to the Einstein metric of a general intersecting configuration. Again, the volume factor  $\sqrt{-\det g_E} = \prod_i D_i^{(p_i+1)/8}$  is identical to the conformal factor  $C^2 = \prod_i D_i^{(p_i+1)/8}$  of the space transverse to all the branes. Indeed, this is a consequence of the harmonic function rule and the corresponding result for individual p-branes. It follows that the Klein-Gordon equation reduces to  $g_E^{\mu\nu} \partial_\mu \partial_\nu \chi = 0$  (in Cartesian coordinates). Multiplication by the conformal factor yields the equation of motion

$$\left[ \sum_{i,j=0}^9 H_i \eta^{ij} \partial_i \partial_j \right] \chi = 0 \quad (14)$$

where  $H_i = \prod_{k_i} D_{k_i}$ . The  $k_i$  run over the indices of the branes that are wrapped around the dimension  $i$ . Specifically the  $\partial_t^2$  term is multiplied by the product of all the harmonics. For example, in the background of Eq. 12, the expression Eq. 14 becomes

$$\left[ D_1 D_2 \square_{k+1} + D_1 (\partial_1^2 + \partial_2^2) + D_2 (\partial_{p+1}^2 + \partial_{p+2}^2) + \Delta \right] \chi = 0 \quad (15)$$

where  $\square_{k+1}$  is the Klein-Gordon operator in the directions parallel to both branes and  $\Delta$  is the Laplacian in the directions transverse to both branes. Other instructive examples include the four-dimensional black holes built from D-branes considered in [27] *e.g.* four 3-branes wrapped around the (123)(345)(146)(256) dimensions of a six-torus. In this case the equation of motion Eq. 14 becomes

$$\begin{aligned} [-D_1 D_2 D_3 D_4 \partial_t^2 &+ D_1 D_2 \partial_3^2 + D_1 D_3 \partial_1^2 + \\ &+ D_1 D_4 \partial_2^2 + D_2 D_3 \partial_4^2 + D_2 D_4 \partial_5^2 + D_3 D_4 \partial_6^2 + \Delta] \chi = 0 \end{aligned} \quad (16)$$

where  $\Delta$  is the Laplacian in three dimensions.

The harmonic functions only depend on the transverse radius  $r$ ; so we can Fourier transform  $\chi$  in the all non-transverse directions. For intersecting branes the effective problem becomes scattering off a potential that includes several different powers of  $1/r$ , instead of a simple Coulomb problem.

### 3 Reflection and Absorption

In this section we will discuss the scattering of neutral massless scalars from configurations of up to four intersecting branes by solving the wave equations derived in the preceding section. We want to study whether the branes can absorb impinging particles because, as discussed in the introduction, inability to absorb a mode implies inability to Hawking radiate into that mode also. To facilitate comparison with the results of Holzhey and Wilczek [16] we toroidally compactify the intersecting branes to four dimensions. This simply has the effect of replacing the harmonic functions  $D = (1 + Q_p/r^{s-2})$  from the previous section with  $D = (1 + Q_p/r)$  where  $r$  is the radial distance in the non-compact dimensions (a numerical factor is absorbed in the definition of  $Q_p$ ). We will consider scattering of scalar fields that are neutral under the

Kaluza-Klein U(1) gauge fields - i.e., fields that are independent of the compactified coordinates. For such neutral fields, we expand in partial waves as  $\chi = R_{l\omega} Y_{lm} e^{-i\omega t}$ , and find:

$$\left[ \frac{\partial}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} - V_{\text{eff}} \right] R_{l\omega} = 0 \quad ; \quad V_{\text{eff}} = -D_1 D_2 D_3 D_4 \omega^2 + \frac{l(l+1)}{r^2} : \quad (17)$$

The functions  $D_i$  have the harmonic form  $1 + \frac{q_i}{r}$  with  $s$  non-vanishing  $q_i$ 's in the case of  $s$  intersecting branes.

### 3.1 One Brane

For a single brane the effective potential is:

$$V_{\text{eff}} = -\omega^2 + \frac{l(l+1)}{r^2} - \frac{\omega^2 Q_1}{r} \quad (18)$$

Then the wave equation in Eq. 17 is formally identical to the Schrödinger equation for a particle of energy  $E = \omega^2$  in an attractive Coulomb potential of charge  $\omega^2 Q_1$ . The exact solution to this problem is known, of course. It is [29]:

$$R_{l\omega} = \frac{C_{\omega l}}{\Gamma(2l+1)} (2\omega r)^l e^{-i\omega r} F\left(\frac{i\omega Q_1}{2} + l + 1, 2l + 2, 2i\omega r\right) \quad (19)$$

where  $F$  is the confluent hypergeometric function and  $C_{\omega l}$  is a normalization factor chosen so that  $\int_0^\infty dr r^2 R_{l\omega'} R_{l\omega} = 2\pi \delta(\omega - \omega')$ . The asymptotic form for large  $r$  is:

$$R_{l\omega} \rightarrow \frac{2}{r} \sin \left( 2\omega r + \frac{Q_1 \omega}{2} \log \omega r - \frac{l\pi}{2} + \delta_l \right) \quad ; \quad \delta_l = \arg \Gamma(l + 1 - i \frac{\omega Q_1}{2}) \quad (20)$$

The relative phase shift between incoming and outgoing waves is:

$$2\Delta_l = -l\pi + 2\delta_l \quad (21)$$

Since  $\Delta_l$  is real, the incoming and outgoing flux at infinity are equal and we can conclude that there is no absorption. In other words, a single extremal p-brane compactified to four dimensions on a 6-torus is unable to absorb impinging particles! The argument summarized here simply formalizes the well-known fact that Coulomb potentials have no absorptive part, even when they are attractive.

### 3.2 Two Branes

For two intersecting branes compactified on a six torus, Eq. 17 gives the effective potential:

$$V_{\text{eff}} = -\omega^2 + \frac{l(l+1) - \omega^2 Q_1 Q_2}{r^2} - \frac{\omega^2 (Q_1 + Q_2)}{r} \quad (22)$$

This is formally identical to the Coulomb potential Eq. 18 with the effective angular momentum given through  $L(L+1) = l(l+1) - \omega^2 Q_1 Q_2$  and the Coulomb constant

modified according to  $Q_1 \rightarrow Q_1 + Q_2$ . It is still  $l$  that is quantized as a positive integer; so the effective angular momentum  $L$  is in general complex. An imaginary part develops for  $Q_1 Q_2 \omega^2 > (l + 1/2)^2$ . Nevertheless the solution is still given by Eq. 19 with suitable replacements in the argument of the hypergeometric function. From the asymptotic expansion Eq. 20 which remains valid for complex of  $l$  we find the relative phase shift between incoming and outgoing waves scattering:

$$2\Delta_l = -L\pi + 2\delta_L \quad \delta_L = \arg \Gamma(L + 1 - i\frac{\omega(Q_1 + Q_2)}{2}) \quad (23)$$

When  $L$  has an imaginary part the phase shift is complex and the incoming flux is not equal to the outgoing flux. This indicates that two intersecting branes absorb impinging particles with frequency  $\omega$  when  $\omega^2 > (l + 1/2)^2/Q_1 Q_2$ .

### 3.3 More Branes

When there are three branes present the effective potential in Eq. 17 acquires a term of the form  $1/r^3$  and when there are four there will also be a  $1/r^4$  term. The corresponding quantum problems can not be solved exactly but an approximate analysis suffices to determine the qualitative behavior [29]. The result is that attractive potentials, behaving as  $r^{-s}$  for small  $r$ , are absorptive for  $s > 2$  and completely elastic for  $s < 2$  while the marginal case with  $s = 2$  depends on the competition between the potential and angular momentum [29]. This implies that both three and four intersecting branes absorb impinging particles regardless of energy.

Rather than repeating the rigorous quantum mechanical analysis we find it instructive to consider the quasi-classical regime of large angular momentum  $l \gg 1$ . Here the wave functions are of the quasi-classical (WKB) form:

$$\chi_0 = \frac{1}{\sqrt{p_r}} \exp(i \int^r p_r) \quad (24)$$

where  $p_r$  is a slowly varying function that satisfies  $-p_r^2 = V_{\text{eff}}$ . The process can then be interpreted as a classical particle subject to the potential  $V_{\text{eff}}$ . It is clear that for  $s = 3, 4$ , the attractive potential  $r^{-s}$  completely dominates the centrifugal barrier and absorption follows for all but the largest  $l$  (corresponding to the classical particle completely missing the black hole). However, for  $s = 2$ , the attractive potential and the centrifugal barrier are of equal importance and the more detailed consideration in Sec. 3.2 is necessary (although the semiclassical analysis happen to give the correct result). In the final case of  $s = 1$  (a single brane), the attractive potential is simply Coulombic. Therefore the centrifugal barrier dominates for large angular momentum, there is a classical turning point, and the impinging particle is completely reflected.

The intuition deriving from the semiclassical approximation apparently fails for the attractive Coulomb problem in the S-wave because here there is no angular momentum barrier and nevertheless the potential reflects, as we saw in the exact treatment in Sec. 3.1. However, by the uncertainty relation, the kinetic energy operator  $p_r^2$  is bounded below by a term of order  $(\frac{1}{2r})^2$ . So quantum uncertainty acts qualitatively

as a classical centrifugal barrier that can be overcome only by potentials that diverges more rapidly than  $1/r^2$  at the center or as  $1/r^2$  with a sufficiently large coefficient. This reconciles the intuitions of this section with the rigorous results presented above.

### 3.4 Comparison With Results In Dilaton Gravity

The results we have derived here can be compared with those of Holzhey and Wilczek for dilaton black holes indexed by the parameter  $a$ . They found absorption impossible for  $a > 1$  and certain for  $a < 1$  while in the case of  $a = 1$  the evidence was inconclusive. However, as described in the introduction, extremal black holes with parameters  $a = \sqrt{3}, 1, \frac{1}{\sqrt{3}}, 0$  are identical to the four dimensional manifestations of 1, 2, 3, and 4 intersecting extremal branes respectively [7, 9, 10]. Our calculation is therefore in perfect harmony with the Holzhey-Wilczek analysis [16]. We find it very satisfying that the marginal case  $a = 1$  directly corresponds to the more familiar marginality of  $r^{-2}$  potentials. It should be noted that gravity has repulsive properties in some contexts, notably in the neighborhood of domain walls [28]. We should therefore emphasize that the effective potential Eq. 17 is always attractive in the S-wave, even for a single brane. As shown in Sec 3.1, absorption by a single compactified brane is prevented by the long range nature of the effective Coulomb-like interaction that governs the radial motion of scalar fields in the p-brane metric, rather than by a repulsive force.

### 3.5 Hawking Radiation From Compactified Branes

The lack of absorption of low-energy modes has consequences for Hawking radiation. Indeed, for  $s = 1$  (a single brane) all finite energy modes are reflected. Now elementary thermodynamics implies that the brane will not radiate into these modes either! In the marginal  $s = 2$  case (two branes), the precise condition for absorption is  $Q_1 Q_2 \omega^2 > (l + \frac{1}{2})^2$ , *i.e.* such black holes exhibit a finite mass gap. Naively Hawking radiation should be perfectly thermal with finite temperature  $T$  where  $T^{-1} = \beta = 4\pi\sqrt{Q_1 Q_2}$  but the greybody factor implied by the gap suppresses the emission amplitude completely for  $\beta\omega < 2\pi(2l + 1)$ . Modes of higher energy can be emitted within this analysis, albeit with exponentially suppressed amplitudes. We may fear that even the smallest amount of neutral radiation inevitably exposes a naked singularity and that two intersecting branes are therefore unstable despite the presence of a gap cutting off low energy radiation. However, the emission of a single high energy mode is sufficient to change the Hawking temperature substantially; so the thermal description is invalid in this regime and a catastrophic fate is probably avoided [17, 30]. Let us also recall that the extremal black holes with  $a = 1$  have a particularly simple description in weakly coupled string theory: they are dual to elementary strings with the right movers in their ground states [31]. These string states are protected by *BPS* saturation and are absolutely stable. Indeed, there are no states in string theory with the same charge but a lower mass. At the present level of treatment the quasi-classical approximation to Hawking radiation does not capture this microscopic picture.



## 4 Non-extremality

The results from the previous sections can be generalized to non-extremal black holes. Effective potentials are of comparable simplicity and facilitate an investigation of the approach to extremality. This enables us to discuss how a single non-extremal compactified brane that is able to absorb particles develops an infinite barrier in the extremal limit.

### 4.1 Effective Potential

Non-extremal versions of any of the extremal configurations of branes from the previous sections are obtained by the introduction of yet another harmonic function  $f = (1 - \mu/r^{s-2})$  where  $s$  is the number of dimensions transverse to all the branes. The non-extremal metric is modified compared to the extremal case by the substitutions [32, 33]:

$$g_{tt} \rightarrow f g_{tt} \quad , \quad g_{rr} \rightarrow f^{-1} g_{rr} \quad (25)$$

This preserves the volume element  $\sqrt{-g_E}$  but the geometry of the space transverse to all the branes changes non-trivially; so the scalar wave equation does not immediately simplify as in previous sections. Note that in the non-extremal case the parameters  $q_i$  of the harmonic functions are non-trivially related to the physical charge as  $Q_i^2 = q_i(\mu + q_i)$ .

Upon toroidal compactification to 4 dimensions the functions  $f$  become  $f = (1 - \mu/r)$  and the metric exhibits a horizon at  $r_+ = \mu$ . The minimally coupled scalar wave equation is:

$$[(-D_1 D_2 D_3 D_4 \partial_t^2 + f \frac{1}{r^2} (\frac{1}{\sin \theta} \partial_\theta \sin \theta \partial_\theta + \partial_\phi^2) + \frac{1}{r^2} f \partial_r (r^2 f \partial_r))] \chi = 0 \quad (26)$$

For brevity we ignored possible dependence of  $\chi$  on the compact dimensions but generality could easily be restored. The *ansatz*  $\chi = \frac{1}{R} Y_{lm} \chi_0 e^{-i\omega t}$  where  $R = r f^{\frac{1}{2}}$  yields:

$$(f^{-2} D_1 D_2 D_3 D_4 \omega^2 - f^{-1} \frac{l(l+1)}{r^2} - \frac{1}{R} R'') \chi_0 + \chi_0'' = 0 \quad (27)$$

where a prime denotes differentiation with respect to  $r$ . Inserting the explicit expression for  $R$  we find the effective potential:

$$V_{\text{eff}} = -f^{-2} D_1 D_2 D_3 D_4 \omega^2 + f^{-1} \frac{l(l+1)}{r^2} - \frac{1}{4} f^{-2} \frac{\mu^2}{r^4} \quad (28)$$

It is instructive to compare this expression with the extremal potential Eq.17. The attractive potential towards the brane is stronger by the factor  $f^{-2}$ . This facilitates fall into the black hole because the centrifugal barrier is only higher by the factor  $f^{-1}$ . There is also an additional attractive potential that only depends on the non-extremality parameter  $\mu$ .

## 4.2 The Approach to Extremality

We now consider a single compactified non-extremal brane with finite temperature and study how the gap in its radiation spectrum switches off the radiation as the extremal limit is attained. We study the S-wave because higher angular momentum radiation is always suppressed relative to the S-wave. Introducing the shifted coordinate  $\rho = r - \mu$  that vanishes at the horizon and expanding in the non-extremality parameter  $\mu$ , the s-wave potential becomes:

$$V_{\text{eff}}^{\text{s-wave}} = -\omega^2(1 + \frac{q}{\rho}) - \frac{1}{\rho^2}\omega^2(q + 2\rho)\mu + \mathcal{O}(\mu^2) \quad (29)$$

This effective potential contains attractive  $1/\rho^2$  and  $1/\rho$  pieces. As shown in Sec. 3.2, such a potential reflects all modes with frequency  $\omega$  where  $\omega^2 q \mu < \frac{1}{4}$ . Approaching extremality  $\mu \rightarrow 0$  we conclude that perturbations with any finite frequency  $\omega$  reflect with certainty! In this precise sense, the brane exhibits an infinite mass gap in the extremal limit and the radiation from the brane is completely suppressed.

A single nonextremal brane is endowed with a finite temperature  $T$ , where  $T^{-1} = \beta = 4\pi\mu(1 + \frac{q}{\mu})^{\frac{1}{2}}$ . In terms of the temperature, the condition for reflection from the non-extremal brane is  $\beta\omega < 2\pi$ . This means that for any given finite  $\mu$  there could be radiation at very high energy energies above this bound. As the extremal limit is approached, the formal temperature diverges and the only modes that can be radiated have diverging energy. These modes have energies that are above the cutoff used to define the semiclassical theory; so radiation into modes reliably described by the Hawking calculation will be completely suppressed in the extremal limit.

## 5 Scattering From Branes in 10 Dimensions

In previous sections we studied branes compactified to four dimensions. However, we expect that a similar analysis applies in more general situation; so we proceed to consider uncompactified extremal p-brane solutions. In 10 dimensions 6-, 5-, and lower branes exhibit infinite, finite, and vanishing temperatures, respectively [33]. This leads to an expectation that the corresponding radial effective potentials behave like  $r^{-1}$ ,  $r^{-2}$ , and  $r^{-s}$  with  $s > 2$  at short distances. To verify this consider minimally coupled scalars that are independent of the directions parallel to the brane. Employing the s-wave *ansatz*  $\chi = y^{-\frac{8-p}{2}}e^{-i\omega t}\chi_0$  the wave equation eq. 10 becomes

$$-\frac{\partial^2}{\partial y^2}\chi_0 - \omega^2(1 + \frac{Q}{y^{7-p}}) + \frac{(8-p)(6-p)}{4y^2}\chi_0 = 0 \quad (30)$$

The first term in the potential is attractive, the second is repulsive. For 6-branes we have an attractive Coulomb potential, as expected. For 5-branes the effective problem involves a  $r^{-2}$  potential, again as expected. Note, however, that details differ from the two intersecting branes in 4 dimensions because of the additional, repulsive  $\frac{3Q}{4y^2}$  potential. Nevertheless, all frequencies below a certain critical one are reflected; so the qualitative features remain unchanged. Similarly, for lower branes, the attractive

$y^{-(7-p)}$  potential clearly dominates at short distances and leads to absorption at all energies. In sum, we find the expected qualitative picture. It appears that the existence of mass gaps protecting extremal objects with finite temperatures is quite a general phenomenon.

## 6 Conclusion

Our results were obtained by studying the dynamics of a minimally coupled spectator scalar. How universal is the qualitative behavior of the appearance of mass gaps? Holzhey and Wilczek [16] found in the context of dilaton gravity that metric, dilaton and gauge field fluctuations all had the same qualitative scattering behavior. In our case it is simple to verify that minimal coupling to either the four dimensional Einstein or string metrics gives the same wave equations as the ones we study. Moreover, momentum in the internal dimensions (*i.e.* charge under the Kaluza-Klein gauge fields) clearly leaves the leading behavior of the effective potential close to the origin unchanged. The qualitative behavior is therefore unmodified, although the precise scattering coefficients will certainly vary for these fields. These examples lead us to believe that the qualitative behavior of repulsion or absorption of low-energy modes by 1,2,3 or 4 intersecting branes is generic.

In recent months, there has been a series of surprising results demonstrating that the classical geometry around near-extremal black holes affects the spectrum of Hawking radiation precisely so that properties of non-perturbative string theory are encoded [34, 35] (for a review see [36]). In this paper, we have tried to reconcile the non-zero temperatures of some classical solutions with their interpretation as stable combinations of D-branes. To do this we have focussed on a curiously strong form of cosmic censorship that not only hides the singularity behind a horizon, but also erects barriers. This renders some compactified p-branes unable to absorb impinging particles of arbitrary finite energy while erecting high classical barriers around other extremal black holes.

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